

Double Universe

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Abstract

We discuss the canonical quantization of non-unitary time evolution in inflating Universe. We consider gravitational wave modes in the FRW metrics in a de Sitter phase and show that the vacuum is a two-mode $SU(1,1)$ squeezed state of thermo field dynamics, thus exhibiting the link between inflationary evolution and thermal properties. In particular we discuss the entropy and the free energy of the system. The state space splits into many unitarily inequivalent representations of the canonical commutation relations parametrized by time t and non-unitary time evolution is described as a trajectory in the space of the representations: the system evolves in time by running over unitarily inequivalent representations. The generator of time evolution is related to the entropy operator. A central ingredient in our discussion is the doubling of the degrees of freedom which turns out to be the bridge to the unified picture of non-unitary time evolution, squeezing and thermal properties in inflating metrics.

1 Introduction

Inflationary Universe scenarios have gained a central position in the interest of cosmologists and, more generally, of physicists interested in quantum aspects of General Relativity. In fact, a phase of primordial inflation explains (or seems to explain) some open problems in General Relativity such as the flatness and horizon problems or the so-called problem of quantum fluctuations [1, 2, 3]. It is therefore of crucial relevance the possibility of describing in quantum field theory (QFT) the non-unitary time evolution implied by inflationary models. The purpose of the present paper is in fact the study of the canonical quantization of inflating time evolution for gravitational wave modes and of their thermal properties.

Quantization is by itself a particularly difficult matter in curved space-time due to the ambiguity in the definition of a privileged frame. The same concept of *vacuum* state is meaningless in the presence of a curved space-time background. The particle production in a gravitational field [4], the Hawking radiation [5] and the generation of gravitational waves from vacuum in an expanding Universe [6, 7, 8] are indeed strictly related with the problem of properly defining the particle number operator in curved space-time.

Even more serious is the problem of quantizing the inflating time evolution in the expanding Universe, since to the problem of the vacuum definition in curved space-time adds up the problem of quantizing the non-unitary time evolution dynamics. Our purpose in this paper is in fact to focus our study on such a last question: the quantization problem related with inflationary time evolution. As we will see, such a problem requires the use of the full richness of QFT, namely of the whole infinite set of unitarily inequivalent representations of the canonical commutation relations. As already stressed in ref. [9] in connection with the problem of the quantization of the matter field in curved space-time, and as clearly stated and showed by Wald [10, 11], QFT in curved space-time does indeed require, to be mathematically well formulated, the use of all possible unitarily inequivalent Hilbert spaces.

Since the pioneering works by Grishchuk, and Grishchuk and Sidorov [6, 7, 8] it is known that relic graviton states are squeezed states of the same kind of squeezed coherent states well known in quantum optics [12, 13]. Two-mode squeezed states for relic gravitons were then discussed in ref. [14], also in connection with the formalism of the entropy associated to linearized perturbations [15]. In this paper our discussion, which will be in fact limited to gravitational wave modes in inflating Universe, will show how the occurrence of the squeezing phenomenon, and in particular of two-mode squeezing, is deeply related with the quantization problem of non-unitary time evolution, indeed; thus revealing a feature not directly emerging from the existing literature on the subject. We note that the dissipative (inflating) term in the gravitational mode equation (see eq. (2.14)) is generally incorporated into the frequency term by using the conformal time variable η [6, 7, 8]; this is a very useful compu-

tational strategy, however our purpose in this paper is to illustrate the subtleties of the canonical quantization for non-unitary time evolution and therefore we must explicitly take care of the dissipative effects. Only in this way the full structure of the state space will be revealed. Moreover, we obtain as an immediate consequence of our result a full set of thermodynamical properties of inflating evolution, including the entropy result of [14]. As a matter of fact, we recover in the present context, the connection with thermal field theory in the formalism set up by Takahashi and Umezawa, called Thermo Field Dynamics (TFD) [16, 17, 18], which anyway underlies many works (even if not referred to) since the paper by Israel [19] on TFD of black holes (see also [20]).

We will reach our conclusions by resorting in a crucial way to results on quantum dissipation obtained in the study of the canonical quantization of the damped harmonic oscillator taken as a prototype of dissipative systems [21]. In such a study, the quantization method for damping oscillator proposed by Feshbach and Tikochinsky [22] in quantum mechanics (QM), was shown to lead to a proper canonical quantization for dissipative systems provided the QFT framework (and not the QM framework, see below) is used. An essential ingredient in the canonical formalism for quantum dissipation is the doubling of the system degrees of freedom, as proposed in fact by Bateman [23] and by Feshbach and Tikochinsky [22].

On the other hand, inflationary metrics also implies time-dependent frequency for the gravitational wave modes, and this leads us to extend the canonical quantization method for non-unitary time evolution so to include the quantization formalism for parametric oscillator [24]. As we will see, and as one should expect [25] (see also [26] with relation to the generalized invariant method), this naturally brings us to squeezed states. The connection with squeezing is also foreseen in the light of the proof of equivalence of damped oscillator states with squeezed states [27].

The emergence of thermodynamical properties and the TFD character of the vacuum state has been already pointed out in the studies of quantum dissipation [21]. And in this respect the doubling of the degrees of freedom, on which also TFD is built upon [16], again has revealed to be a crucially useful tool, rich of physical content.

One reason why in TFD thermal aspects of QFT may be naturally treated is that in such a formalism all the statistical properties arise without any superimposed condition and all the states (included the vacuum) have a thermal valence. This is obtained by doubling the system degrees of freedom. Such a doubling allows to take into account not only the system under consideration but also the thermal bath (or environment) in which it is embedded. From an operatorial and algebraic point of view the doubling of the degrees of freedom is deeply related with the C^* -algebraic structure of the theory [28] (in particular with the Gel'fand- Naimark-Segal (GNS) construction [29]); more recently it has been pointed out that the algebraic structure of TFD is fully included in the quantum deformation of the associated Hopf algebra,

the doubling being related to the coproduct and the quantum deformation parameter being related to the non-zero value of the temperature [30].

In the doubling formalism, developed for the damping phenomena in QFT [21] (see also [31, 32, 33]) and here adopted for the present discussion, the ground state (vacuum) and the corresponding Fock space are labelled by the time t : $|0(t)\rangle$ and \mathcal{H}_t , respectively. In the infinite volume limit, for different times, $t \neq t'$, $|0(t)\rangle$ turns out to be orthogonal to $|0(t')\rangle$ and \mathcal{H}_t unitarily inequivalent to $\mathcal{H}_{t'}$. In this way one obtains the parametrization by t of infinitely many representations \mathcal{H}_t of the canonical commutation relations. The non-unitary character of time evolution implied by dissipation finds its description in the non-unitary equivalence among the \mathcal{H}_t representations at different t 's: the system state space thus splits into many unitarily inequivalent representations. We call the collection of the \mathcal{H}_t representations the "representation space" $\{\mathcal{H}_t\}$. Time evolution is then described as a trajectory in the space of the representations: the system evolves in time by running over unitarily inequivalent representations. We also show that the generator responsible for non-unitary time evolution is related to the entropy operator. This fact should not be surprising (see [31, 21]) since dissipation (inflation) involves irreversibility in time evolution (*the arrow of time*). As a matter of fact, the system states are recognized to be SU(1,1) coherent states of the same kind of the thermal states of TFD, thus recovering a full set of thermal properties. The emerging picture of the canonical quantization scheme we obtain is thus a unified view of many features of inflationary evolution, including coherence, two-mode squeezing, entropy and vacuum thermal properties. It explicitly shows how the algebraic construction utilizing the infinitely many unitarily inequivalent representations works; in some sense it is the GNS construction of C^* algebra "at work" [10, 11, 29].

We perform the analysis of inflating metrics by using the quasi-linear approximation. In the de Donder gauge condition, Einstein equations lead, in the inflating case of Friedmann-Robertson-Walker (FRW) metrics, to the damped harmonic oscillator equation for the partial waves of the field $h_{\mu\nu}$ [6, 7, 8] (for this reason, even when in our discussion we use the word dissipation we actually always refer to the inflating scenario). We then proceed with the quantization method for the damped oscillator mentioned above. The doubling of the degrees of freedom reflects itself on the metric structure in such a way we have the doubling of the $h_{\mu\nu}$ partial waves. In this sense we speak of "double Universe".

The physical interpretation is that the doubled degrees of freedom introduce the *complement* to the inflating system, thus *closing* it, as required by the canonical quantization procedure. This points to the root of the mathematical difficulty in the canonical quantization of inflating evolution: we show in fact that non-unitary time evolution cannot be properly quantized if a single representation of the canonical commutation relations is available (as it happens in QM). However, the von Neumann theorem of QM does not hold in QFT, due to the infinite number of degrees of

freedom, and the existence of infinitely many unitarily inequivalent representations of the canonical commutation relations is allowed. Then, the quantization of inflating systems may be performed by taking advantage of the full space of the representations, and this is achieved by closing the system (by doubling the degrees of freedom) and by allowing evolution by tunneling *through* the representations. As we said above, the emerging vacuum structure is the one of the squeezed coherent states. Squeezing has thus a deep dynamical origin and is related with the intrinsic group theory properties of the inflating evolution. The condensate vacuum structure is such that the difference between the number of relic graviton modes and of their doubled modes is constant in time (creation and annihilation occurs always in *pairs*) and the doubled mode can be interpreted as the *hole* for graviton mode. This reminds us of the similar situation occurring in the matter field particle creation in curved space-time background [9].

It is also interesting to observe that the damping term in the wave mode evolution equation is actually a manifestation of the curvature represented by the time-dependent inflating metrics; and that complementing the system in order to proceed to canonical quantization is actually equivalent to *shielding* the curvature effects due to such an inflating time-dependence. This is related with the *gauge* structure of quantum dissipation [36] and of TFD [30, 34, 35].

The paper is organized as follows. In Sec. 2 we introduce general, well known features of the quasi-linear approximation leading in the inflating case to the damped parametric oscillator equation for the $h_{\mu\nu}$ wave modes. In Sec. 3 we start the discussion of the system quantization, also showing there how the doubling of the degrees of freedom works in order to complement the system. In Sec. 4 we obtain the Hamiltonian spectrum by the method of the spectrum generating algebra and exhibit the theory vacuum structure, its time evolution and its two-mode squeezing character. We also show that the QM framework is not adequate in view of the vacuum instability. In Sec. 5 we cure this pathology by moving to the QFT framework and we discuss the evolution as trajectories *over* unitarily inequivalent representations. Entropy, free energy and the first principle of thermodynamics are discussed in Sec. 6. It is interesting to remark that entropy appears in our formalism as the generator of the non-unitary time evolution and at the same time as the free energy response to temperature variations; moreover, heat dissipation is related to variations in the condensate structure of the vacuum.

In this paper, since our interest is mainly focused on the possibility of setting up a canonical quantization scheme for inflating evolution exhibiting at once general features as squeezing and thermal properties, we do not consider more specific model features, or renormalization problems, neither we study symmetry restoration mechanisms due to thermal effects. Concluding remarks are reported in Sec. 7.

2 Inflating Universe

In the four dimensional space-time $x^\mu = \{x_0 = ct, x^i\}$, $i = 1, 2, 3$, we consider the so-called linear approximation where one decomposes the metrics $g_{\mu\nu}$ as

$$g_{\mu\nu} = g_{\mu\nu}^0 + h_{\mu\nu}. \quad (2.1)$$

When one chooses the flat background metrics

$$g_{\mu\nu}^0 = \eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (2.2)$$

as customary one defines

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h, \quad (2.3)$$

with $h = h^\mu_\mu$, which transforms as

$$\bar{h}_{\mu\nu} \rightarrow \bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} - \partial_\mu \varsigma_\nu - \partial_\nu \varsigma_\mu + \eta_{\mu\nu} \partial_\lambda \varsigma^\lambda \quad (2.4)$$

when

$$x^\mu \rightarrow x'^\mu = x^\mu + \varsigma^\mu(x). \quad (2.5)$$

In the same way as in Q.E.D., the (de Donder) gauge condition

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \quad (2.6)$$

is satisfied provided the class of gauge functions ς^λ solutions of

$$\square \varsigma^\lambda = 0 \quad (2.7)$$

is adopted. In the gauge (2.6) the Einstein equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0 \quad (2.8)$$

give

$$\square \bar{h}^{\mu\nu} = 0. \quad (2.9)$$

The field $\bar{h}^{\mu\nu}$ is then decomposed in partial waves

$$\bar{h}^{\mu\nu} = \sum_\lambda \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} e_{\mathbf{k}\lambda}^{\mu\nu} \{u_{\mathbf{k}\lambda}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + u_{\mathbf{k}\lambda}^\dagger(t)e^{-i\mathbf{k}\cdot\mathbf{x}}\} \quad (2.10)$$

with $k \equiv (k_0 = \omega = ck, \mathbf{k})$. The wave function $u_{\mathbf{k}\lambda}(t)$ satisfies the simple harmonic oscillator equation

$$\frac{d^2}{dt^2}u_{\mathbf{k}\lambda}(t) + \omega^2 u_{\mathbf{k}\lambda}(t) = 0 \quad . \quad (2.11)$$

When, instead of $\eta_{\mu\nu}$, the metrics

$$g_{\mu\nu}^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^2(t) & 0 & 0 \\ 0 & 0 & -a^2(t) & 0 \\ 0 & 0 & 0 & -a^2(t) \end{pmatrix} \quad (2.12)$$

with

$$a(t) = a_0 e^{\frac{1}{3}Ht}, \quad (2.13)$$

and $H = \text{const.} = \frac{3\dot{a}(t)}{a(t)}$ (the Hubble constant), is adopted, $\bar{h}^{\mu\nu}$ may be still decomposed in partial waves as in (2.10); however, in such a case one obtains the equation for the parametric (frequency time-dependent) damped harmonic oscillator [1, 6, 7, 8] (see also [37])

$$\ddot{u}(t) + H \dot{u}(t) + \omega^2(t)u(t) = 0 \quad (2.14)$$

with

$$\omega^2(t) = \frac{k_3^2 c^2}{a^2(t)} \quad . \quad (2.15)$$

In Eq.(2.14) we have used $u(t) \equiv u_{\mathbf{k}\lambda}(t)$. In the Minkowsky space-time ω is constant in time, but when the Universe expands, ω is time-dependent $\omega = \omega(t)$. This can be seen as a generalization of the Doppler effect.

The dissipative (inflating) term $H \dot{u}$ in eq. (2.14) is generally incorporated into the frequency term by using the conformal time variable η [6, 7, 8]; as observed in the introduction such a computational strategy is very useful in the phenomenological approach, however our purpose in this paper is to illustrate the subtelties of the canonical quantization for non-unitary time evolution and therefore we must explicitly take care of the dissipative term in eq. (2.14). Only in this way the full structure of the state space will be revealed.

It is interesting to remark that eq.(2.14) with $a(t)$, H and $\omega(t)$ given by (2.13) and (2.15) is also known as Hill-type equation [38]. By setting $\xi = \epsilon z$, $\epsilon = \frac{3k_3 c}{a_0 H}$, $z = e^{-\frac{H}{3}t}$ and $u = z^2 V$ we can easily show that it can be written in the form of the spherical Bessel equation

$$\xi^2 V_{\xi\xi} + 2\xi V_{\xi} + (\xi^2 - 2) V = 0 \quad . \quad (2.16)$$

One particular solution of eq.(2.16) is indeed the spherical Bessel function of the first kind

$$j_1(\xi) = \frac{\sin \xi}{\xi^2} - \frac{\cos \xi}{\xi} \quad , \quad (2.17)$$

i.e.

$$u(t) = \frac{1}{\epsilon^2} \left[\sin(\epsilon e^{-\frac{H}{3}t}) - \epsilon e^{-\frac{H}{3}t} \cos(\epsilon e^{-\frac{H}{3}t}) \right] , \quad (2.18)$$

which in fact is solution of (2.14).

3 Quantization of inflating time evolution

In ref.[21] it has been discussed the quantization of the one dimensional damped harmonic oscillator with constant frequency, i.e. of eq. (2.14) with $\omega^2(t) = \text{const.}$, and it has been shown that the canonical quantization can be properly performed by doubling the degrees of freedom of the system and by working in the QFT framework. The physical reason to double the degrees of freedom of the dissipative (damped) system relies in the fact that one must work with closed systems as required by the canonical quantization formalism. On the other hand, since the system states split into unitarily inequivalent representations of the canonical commutation relations [21], one is forced to use QFT where infinitely unitarily inequivalent representations indeed exist (QM is not adequate due to the Von Neumann theorem that states that all the representations are unitarily equivalent for systems with finite number of degrees of freedom; see the discussion in [21]). Therefore, in order to perform the canonical quantization of the oscillator (2.14), according to [21] we consider the double oscillator system

$$\ddot{u} + H \dot{u} + \omega^2(t)u = 0 , \quad (3.1)$$

$$\ddot{v} - H \dot{v} + \omega^2(t)v = 0 , \quad (3.2)$$

and in this sense we speak of a "double Universe". We observe indeed that in the same way eq. (3.1) is implied by the inflating metrics, eq. (3.2) for the oscillator v can be associated to the "deflating" metrics

$$\tilde{g}_{\mu\nu}^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{a^2(t)} & 0 & 0 \\ 0 & 0 & -\frac{1}{a^2(t)} & 0 \\ 0 & 0 & 0 & -\frac{1}{a^2(t)} \end{pmatrix} \quad (3.3)$$

with $a(t)$ given by (2.13). We thus may think of the oscillator v as associated to the "deflating" Universe, complementary to the inflating one. Note that $\tilde{g}_{\mu\nu}^0 = (g_{\mu\nu}^0)^{-1}$ and that the D'Alembert operator leading to (3.2) is given by $-\sqrt{-g}\partial^\mu \frac{1}{\sqrt{-g}}g_{\mu\nu}\partial^\nu$ with $g \equiv \det(g_{\mu\nu})$.

In order to proceed in our discussion it is now convenient to write down the Lagrangian in terms of u and v modes from which eqs.(3.1), (3.2) are directly obtained:

$$L = \dot{u}\dot{v} + \frac{1}{2}H(u\dot{v} - v\dot{u}) - \omega^2(t)uv . \quad (3.4)$$

The canonical momenta are

$$p_u = \frac{\partial L}{\partial \dot{u}} = \dot{v} - \frac{1}{2}Hv , \quad (3.5)$$

$$p_v = \frac{\partial L}{\partial \dot{v}} = \dot{u} + \frac{1}{2}Hu , \quad (3.6)$$

and the Hamiltonian is

$$\mathcal{H} = p_u \dot{u} + p_v \dot{v} - L = p_u p_v + \frac{1}{2}H(vp_v - up_u) + \left(\omega^2(t) - \frac{H^2}{4} \right) uv \quad . \quad (3.7)$$

We will put $\Omega(t) \equiv \left[\left(\omega^2(t) - \frac{H^2}{4} \right) \right]^{\frac{1}{2}}$, which we will get real, i.e. $\omega^2(t) > \frac{H^2}{4}$: i.e. $0 \leq t < \frac{3}{H} \ln\left(\frac{2k_3 c}{a_0 H}\right)$, with $\frac{2k_3 c}{a_0 H} \geq 1$; the same reality condition is however also fulfilled for $k_3 \geq \frac{a_0 H}{2c} e^{\frac{H}{3}t}$ which tells us that as time flows the reality condition excludes long wave modes, which in turn acts as an intrinsic infrared cut-off, very welcome, as we will see, for the well-definiteness of the operator fields. We therefore will adopt such a last constraint in our subsequent discussion. See also [1] for a discussion of time domains in inflating models.

It is interesting to observe that $v = ue^{Ht}$ is solution of (3.2) and that by setting $u(t) = \frac{1}{\sqrt{2}}r(t)e^{\frac{-Ht}{2}}$ and $v(t) = \frac{1}{\sqrt{2}}r(t)e^{\frac{Ht}{2}}$ the system of equations (3.1) and (3.2) is equivalent to the equation

$$\ddot{r} + \Omega^2(t)r = 0 , \quad (3.8)$$

which is the equation for the parametric oscillator $r(t)$ (see also [36]). This further clarifies the meaning of the doubling of the u oscillator: the $u - v$ system is a non-inflating (and non-deflating) system. This is why it is now possible to set up the canonical quantization scheme.

We introduce indeed as usual the commutators

$$[u, p_u] = i\hbar = [v, p_v] \quad , \quad [u, v] = 0 = [p_u, p_v] \quad , \quad (3.9)$$

and it turns out to be convenient to introduce the variables U and V by the transformations

$$U(t) = \frac{u(t) + v(t)}{\sqrt{2}}, \quad V(t) = \frac{u(t) - v(t)}{\sqrt{2}} \quad , \quad (3.10)$$

which preserve the commutation relations (3.9):

$$[U, p_U] = i\hbar = [V, p_V] \quad , \quad [U, V] = 0 = [p_U, p_V] \quad . \quad (3.11)$$

In terms of the variables U and V it now appears that we are dealing with the decomposition of the parametric oscillator $r(t)$ on the *hyperbolic* plane (i.e. in the pseudoeuclidean metrics): $r^2(t) = U^2(t) - V^2(t)$.

The Lagrangian (3.4) is rewritten in terms of U and V as

$$L = L_{0,U} - L_{0,V} + \frac{H}{2}(\dot{U}V - \dot{V}U) \quad , \quad (3.12)$$

with

$$L_{0,U} = \frac{1}{2}\dot{U}^2 - \frac{\omega^2(t)}{2}U^2 \quad , \quad L_{0,V} = \frac{1}{2}\dot{V}^2 - \frac{\omega^2(t)}{2}V^2 \quad . \quad (3.13)$$

The associate momenta are

$$p_U = \dot{U} + \frac{H}{2}V \quad , \quad p_V = -\dot{V} - \frac{H}{2}U \quad , \quad (3.14)$$

and the motion equations corresponding to the set (3.1)- (3.2) are

$$\ddot{U} + H\dot{V} + \omega^2(t)U = 0 \quad , \quad (3.15)$$

$$\ddot{V} + H\dot{U} + \omega^2(t)V = 0 \quad . \quad (3.16)$$

The Hamiltonian (3.7) becomes

$$\mathcal{H} = \mathcal{H}_U - \mathcal{H}_V = \frac{1}{2}(p_U - \frac{H}{2}V)^2 + \frac{\omega^2(t)}{2}U^2 - \frac{1}{2}(p_V + \frac{H}{2}U)^2 - \frac{\omega^2(t)}{2}V^2 \quad . \quad (3.17)$$

Eq.(3.12) shows that the inflation (or dissipative) H -term actually acts as a coupling between the oscillators U and V and produces a correction to the kinetic energy for both oscillators (cf.eq.(3.17)). In the following we will see that the group structure underlying the Hamiltonian (3.17) is the one of $SU(1,1)$. The occurrence of the minus sign between $L_{0,U}$ and $L_{0,V}$ and H_U and H_V , respectively, is in fact directly related with the $SU(1,1)$ group structure.

As already observed above, our problem also involves the quantization of the parametric oscillator. We then proceed to the quantization of (3.17) by resorting to the quantization method of the parametric harmonic oscillator described in ref. [24] and to the quantization method for the damped oscillator of ref.[21]. To this aim it is better to write the Hamiltonian (3.17) in the form:

$$\mathcal{H} = \frac{1}{2}p_U^2 + \frac{1}{2}\Omega^2(t)U^2 - \frac{1}{2}p_V^2 - \frac{1}{2}\Omega^2(t)V^2 - \frac{H}{2}(p_U V + p_V U) \quad . \quad (3.18)$$

We introduce the annihilation and creation operators:

$$A = \frac{1}{\sqrt{2}} \left(\frac{p_U}{\sqrt{\hbar\omega_0}} - iU\sqrt{\frac{\omega_0}{\hbar}} \right) , \quad A^\dagger = \frac{1}{\sqrt{2}} \left(\frac{p_U}{\sqrt{\hbar\omega_0}} + iU\sqrt{\frac{\omega_0}{\hbar}} \right) , \quad (3.19)$$

$$B = \frac{1}{\sqrt{2}} \left(\frac{p_V}{\sqrt{\hbar\omega_0}} - iV\sqrt{\frac{\omega_0}{\hbar}} \right) , \quad B^\dagger = \frac{1}{\sqrt{2}} \left(\frac{p_V}{\sqrt{\hbar\omega_0}} + iV\sqrt{\frac{\omega_0}{\hbar}} \right) , \quad (3.20)$$

with commutation relations

$$[A, A^\dagger] = 1 = [B, B^\dagger], \quad [A, B] = 0, \quad [A^\dagger, B^\dagger] = 0, \quad (3.21)$$

and all other commutators equal to zero. In eqs. (3.19), (3.20) ω_0 denotes an arbitrary frequency introduced according to the quantization procedure for parametric oscillator discussed in ref. [24]. By putting $x = U, V$, we indeed have (cf. eq.(3.18)):

$$\frac{1}{2}p_x^2 + \frac{1}{2}\Omega^2(t)x^2 = \Omega_0 K_0^x - \Omega_1 K_1^x \quad (3.22)$$

with

$$\Omega_{0,1} = \omega_0 \left(\frac{\Omega^2(t)}{\omega_0^2} \pm 1 \right), \quad (3.23)$$

and

$$K_{0,1}^x = \frac{1}{2\omega_0} \left(\frac{p_x^2}{2} \pm \frac{x^2}{2}\omega_0^2 \right), \quad (3.24)$$

which together with

$$K_2^x = \frac{1}{4}(p_x x + x p_x) \quad (3.25)$$

close the $su(1, 1)$ algebra:

$$[K_1^x, K_2^x] = -iK_0^x, \quad [K_2^x, K_0^x] = iK_1^x, \quad [K_0^x, K_1^x] = iK_2^x. \quad (3.26)$$

Of course, $[K_i^x, K_j^{x'}] = 0$ for any i, j and $x \neq x'$. By using eqs. (3.19)-(3.20) and eqs. (3.24) and (3.25) we introduce then the operators

$$\begin{aligned} K_0 &= \frac{1}{2}(A^\dagger A - B^\dagger B) & K_1 &= \frac{1}{4}[(A^2 + A^{\dagger 2}) - (B^2 + B^{\dagger 2})], \\ K_2 &= i\frac{1}{4}[(A^2 - A^{\dagger 2}) + (B^2 - B^{\dagger 2})], \end{aligned} \quad (3.27)$$

which also close the algebra $su(1, 1)$:

$$[K_1, K_2] = -iK_0, \quad [K_2, K_0] = iK_1, \quad [K_0, K_1] = iK_2, \quad (3.28)$$

We further remark that

$$J_+ = A^\dagger B^\dagger, \quad J_- = AB, \quad J_0 = \frac{1}{2}(A^\dagger A + B^\dagger B + 1), \quad (3.29)$$

with $J_1 = \frac{1}{2}(J_+ + J_-)$ and $J_2 = -\frac{i}{2}(J_+ - J_-)$, also close the $su(1, 1)$ algebra. Note that

$$2iJ_2 = A^\dagger B^\dagger - AB \quad (3.30)$$

commutes with each K_i , $i = 0, 1, 2$ (eqs. (3.27)), and that

$$\mathcal{C} \equiv \frac{1}{2}(A^\dagger A - B^\dagger B) = K_0 \quad (3.31)$$

commutes with each J_i , $i = 0, +, -$ (eqs. (3.29)). $2iJ_2$ and \mathcal{C} are indeed (related to) the Casimir operators for the algebras of generators (3.27) and (3.29), respectively.

In terms of A and B the Hamiltonian (3.18) is finally written as

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{I_1} + \mathcal{H}_{I_2} \quad (3.32)$$

$$\mathcal{H}_0 = \frac{1}{2}\hbar\Omega_0(t)(A^\dagger A - B^\dagger B) = \hbar\Omega_0(t)\mathcal{C} = \hbar\Omega_0(t)K_0, \quad (3.33)$$

$$\mathcal{H}_{I_1} = -\frac{1}{4}\hbar\Omega_1(t) \left[(A^2 + A^{\dagger 2}) - (B^2 + B^{\dagger 2}) \right] = -\hbar\Omega_1(t)K_1, \quad (3.34)$$

$$\mathcal{H}_{I_2} = i\Gamma\hbar (A^\dagger B^\dagger - AB) = i\hbar\Gamma(J_+ - J_-). \quad (3.35)$$

with $\Gamma \equiv \frac{\hbar}{2}$.

We note that for any t

$$[\mathcal{H}_0, \mathcal{H}_{I_2}] = 0 = [\mathcal{H}_{I_1}, \mathcal{H}_{I_2}], \quad (3.36)$$

which guaranties that under time evolution the minus sign appearing in \mathcal{H}_0 is not harmful (i.e., once one starts with a positive definite Hamiltonian it remains lower bounded).

Finally, we recall that the A and B operators (and all other operators) as well as other quantities, e.g. $\omega(t)$, actually are dependent on the momentum \mathbf{k} (and on other degrees of freedom) and thus our formulas should be understood as carrying such a \mathbf{k} labels which we have been omitting for simplicity. For instance the commutators (3.21) are indeed to be understood as

$$[A_{\mathbf{k}}, A_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{k}, \mathbf{k}'} = [B_{\mathbf{k}}, B_{\mathbf{k}'}^\dagger], \quad [A_{\mathbf{k}}, B_{\mathbf{k}'}] = 0, \quad [A_{\mathbf{k}}^\dagger, B_{\mathbf{k}'}^\dagger] = 0. \quad (3.37)$$

We also observe that the operators $K_{i, \mathbf{k}}$, $i = 0, 1, 2$ (or $i = 0, +, -$), for fixed \mathbf{k} close the algebra $su_{\mathbf{k}}(1, 1)$ and that they commute for any i, j for $\mathbf{k} \neq \mathbf{k}'$.

Our next task is to study the Hilbert space structure and this will be done in the following section.

4 The vacuum structure

Again, for simplicity, we will omit the \mathbf{k} indeces whenever no misunderstanding arises. They will be restored at the end.

In order to study the eigenstates of the Hamiltonian (3.32), we consider the set $\{|n_A, n_B\rangle\}$ of simultaneous eigenvectors of $A^\dagger A$ and $B^\dagger B$, with n_A, n_B non-negative integers. These are eigenstates of \mathcal{H}_0 with eigenvalues $\frac{1}{2}\hbar\Omega_0(t)(n_A - n_B)$ for any t . The eigenstates of \mathcal{H}_{I_2} can be written in the standard basis, in terms of the eigenstates of $(J_3 - \frac{1}{2})$ in the representation labelled by the value $j \in Z_{\frac{1}{2}}$ of \mathcal{C} , $\{|j, m\rangle; m \geq |j|\}$:

$$\mathcal{C}|j, m\rangle = j|j, m\rangle \quad , \quad j = \frac{1}{2}(n_A - n_B) \quad ; \quad (4.1)$$

$$\left(J_3 - \frac{1}{2}\right)|j, m\rangle = m|j, m\rangle \quad , \quad m = \frac{1}{2}(n_A + n_B) \quad . \quad (4.2)$$

By using the $su(1, 1)$ algebra

$$[J_+, J_-] = -2J_3 \quad , \quad [J_3, J_\pm] = \pm J_\pm \quad , \quad (4.3)$$

one can show that the kets $|\psi_{j,m}\rangle \equiv e^{(+\frac{\pi}{2}J_1)}|j, m\rangle$ satisfy indeed the equation $J_2|\psi_{j,m}\rangle = \mu|\psi_{j,m}\rangle$ with pure imaginary $\mu \equiv i\left(m + \frac{1}{2}\right)$.

Notice that although J_2 appears to be hermitian, it has pure imaginary discrete spectrum in $|\psi_{j,m}\rangle$. This apparent contradiction is related with the well known [22, 39] fact that the states $|\psi_{j,m}\rangle$ are not normalizable and the transformation generating $|\psi_{j,m}\rangle$ from $|j, m\rangle$ is a non-unitary transformation in $SU(1, 1)$ (it is not a proper rotation in $SU(1, 1)$, but is rather a pseudorotation in $SL(2, \mathbb{C})$ [21]). In other words $|\psi_{j,m}\rangle$ does not provide a unitary irreducible representation (*UIR*), consistently with the fact [40] that in any *UIR* of $SU(1, 1)$ J_2 should have purely continuous and real spectrum (which does not happen in the case of $|\psi_{j,m}\rangle$). However, *UIR* with continuous and real spectrum are not adequate for the description of dissipating and inflating phenomena and therefore they are of no help to us.

By following refs. [21, 22] we can bypass this difficulty by introducing in the Hilbert space a new metric with a suitable inner product in such a way that $|\psi_{j,m}\rangle$ has a finite norm. To this aim we consider the antiunitary operation \mathcal{T} under which $(A, B) \xrightarrow{\mathcal{T}} (-A^\dagger, -B^\dagger)$ and introduce the conjugation $\langle \psi_{j,m} | \equiv [\mathcal{T}|\psi_{j,m}\rangle]^\dagger$, with $\mathcal{T}|\psi_{j,m}\rangle \equiv |\psi_{j, -(m+1)}\rangle$. The hermitian of $J_2|\psi_{j,m}\rangle = \mu|\psi_{j,m}\rangle$, $\mu \equiv i\left(m + \frac{1}{2}\right)$, is now

$$\langle \psi_{j, -(m+1)} | J_2 = \mu_{\mathcal{T}} \langle \psi_{j, -(m+1)} | \quad , \quad \mu_{\mathcal{T}} = -i \left[-(m+1) + \frac{1}{2} \right] = -\mu^* = \mu \quad . \quad (4.4)$$

We are left with the discussion of the eigenstates of \mathcal{H}_{I_1} . We can "rotate away" [41] \mathcal{H}_{I_1} by using the transformation $\mathcal{H} \rightarrow \mathcal{H}' \equiv e^{i\theta(t)K_2} \mathcal{H} e^{-i\theta(t)K_2}$ with $\tanh \theta(t) = -\frac{\Omega_1(t)}{\Omega_0(t)}$ at any t . We obtain

$$\mathcal{H}' \equiv e^{i\theta(t)K_2} \mathcal{H} e^{-i\theta(t)K_2} = \mathcal{H}'_0 + \mathcal{H}_{I_2} \quad , \quad \tanh \theta(t) = -\frac{\Omega_1(t)}{\Omega_0(t)} \quad , \quad (4.5)$$

with

$$\mathcal{H}'_0 = \hbar\Omega(t)(A^\dagger A - B^\dagger B) \quad (4.6)$$

$$[\mathcal{H}'_0, \mathcal{H}_{I_2}] = 0. \quad (4.7)$$

Here we have used the algebra (3.28), eq. (3.23) and the fact that \mathcal{H}_{I_2} commutes with K_2 . Note that when the \mathbf{k} indices are restored we have $\theta(t) \equiv \theta_{\mathbf{k}}(t)$. Also note that the choice $\tanh \theta(t) = -\frac{\Omega_1(t)}{\Omega_0(t)}$ is allowed since the modulus of $(-\frac{\Omega_1(t)}{\Omega_0(t)})$ is at most equal to one (for any \mathbf{k} and any t).

In conclusion, the eigenstates of the Hamiltonian \mathcal{H} at t , eq. (3.32), are states of type $e^{-i\theta(t)K_2}|\psi_{j,m} >$:

$$\begin{aligned} \mathcal{H}e^{-i\theta(t)K_2}|\psi_{j,m} > &= e^{-i\theta(t)K_2}e^{i\theta(t)K_2}\mathcal{H}e^{-i\theta(t)K_2}|\psi_{j,m} > = \\ e^{-i\theta(t)K_2}(\mathcal{H}'_0 + \mathcal{H}_{I_2})|\psi_{j,m} > &= (\hbar\Omega(t)(n_A - n_B) - i\hbar\Gamma(n_A + n_B + 1))e^{-i\theta(t)K_2}|\psi_{j,m} >, \end{aligned} \quad (4.8)$$

where we have been using the algebras (3.28), (4.3) and the commuting properties of the related Casimir operators. We note that $\Omega(t)$ appears to be the common frequency of the two oscillators A and B . In conclusion, the spectrum of \mathcal{H} is determined by using the so-called method of the spectrum generating algebras [24, 41].

The solution to the Schrödinger equation can be given with reference to the initial time pure state $|j, m_0 >$ (see refs. [21, 22]). When in particular, the initial state, say at arbitrary initial time t_0 , is the *vacuum* for \mathcal{H}'_0 , i.e. $|n_A = 0, n_B = 0 > \equiv |0 >$, with $A|0 > = 0 = B|0 >$ (i.e. $j = 0, m_0 = 0$ for any \mathbf{k}), the state

$$|0(\theta(t_0)) > = e^{-i\theta(t_0)K_2}|0 >, \quad (4.9)$$

at t_0 (and for given \mathbf{k}), is the zero energy eigenstate (the *vacuum*) of $\mathcal{H}_0 + \mathcal{H}_{I_1}$ at t_0 :

$$(\mathcal{H}_0 + \mathcal{H}_{I_1})|_{t_0}|0(\theta(t_0)) > = e^{-i\theta(t_0)K_2}\mathcal{H}'_0|0 > = 0 \text{ for any arbitrary } t_0. \quad (4.10)$$

Notice that $\mathcal{H}'_0|0 > = 0$ for any t , but that $(\mathcal{H}_0 + \mathcal{H}_{I_1})|0 > \neq 0$ and $\mathcal{H}'_0|0(\theta(t_0)) > \neq 0$ for any t . However, expectation values of \mathcal{H}'_0 and of $(\mathcal{H}_0 + \mathcal{H}_{I_1})$ in $|0 >$ and in $|0(\theta(t_0)) >$ are all zero at any t (see also below).

The state $|0(\theta(t)) >$ is a generalized $su(1,1)$ coherent state, appearing, as well known, in the study of the parametric excitations of the quantum oscillator [24].

In the following for simplicity we will put $t_0 = 0$ and set $\theta(t_0 = 0) \equiv \theta$ and $|0(\theta(t_0 = 0)) > \equiv |0(\theta) >$.

Since A and B are commuting operators we can write

$$\exp(-i\theta K_2) = \exp\left(\frac{\theta}{4}(A^2 - A^{\dagger 2})\right) \exp\left(\frac{\theta}{4}(B^2 - B^{\dagger 2})\right), \quad (4.11)$$

i.e. $\exp(-i\theta K_2)$ can be factorized as the product of two (commuting) single-mode squeezing generators and for this reason we will refer to the state $|0(\theta) >$ as to

the squeezed vacuum (at this level actually it is not, strictly speaking, a squeezed state since squeezed states are obtained by applying the squeezing generator to a (Glauber-type) coherent state).

Time evolution of the squeezed vacuum $|0(\theta)\rangle$ is given by

$$|0(\theta, t)\rangle = \exp\left(-it\frac{\mathcal{H}}{\hbar}\right)|0(\theta)\rangle = \exp\left(-it\frac{\mathcal{H}_{I_2}}{\hbar}\right)|0(\theta)\rangle, \quad (4.12)$$

due to (3.36) and to (4.10). Notice that in eq.(4.12) \mathcal{H} denotes the Hamiltonian at time $t_0 = 0$, *i.e.* the Hamiltonian (3.32) with $\Omega_0(t_0 = 0)$ and $\Omega_1(t_0 = 0)$. Note that (4.12) can be written also as

$$|0(\theta, t)\rangle = \exp(-i\theta K_2) \exp\left(-it\frac{\mathcal{H}_{I_2}}{\hbar}\right)|0\rangle = \exp(-i\theta K_2)|0(t)\rangle. \quad (4.13)$$

We can now better appreciate the advantage of explicitly taking care of the dissipative term in eq. (2.14): the contributions from the non-unitary evolution term and from the frequency term are now separated and explicitly shown in eqs. (4.12) and (4.13).

Let us study $|0(\theta, t)\rangle$ given by (4.12). We observe that the operators A and B transform under $\exp(-i\theta K_2)$ (for any given \mathbf{k}) as

$$A \mapsto A(\theta) = e^{-i\theta K_2} A e^{i\theta K_2} = A \cosh\left(\frac{1}{2}\theta\right) + A^\dagger \sinh\left(\frac{1}{2}\theta\right), \quad (4.14)$$

$$B \mapsto B(\theta) = e^{-i\theta K_2} B e^{i\theta K_2} = B \cosh\left(\frac{1}{2}\theta\right) + B^\dagger \sinh\left(\frac{1}{2}\theta\right). \quad (4.15)$$

These transformations are the well known squeezing transformations which preserve the commutation relations (3.21) (and (3.37)). One has

$$A(\theta)|0(\theta)\rangle = 0 = B(\theta)|0(\theta)\rangle, \quad (4.16)$$

and the number of modes of type A in the state $|0(\theta)\rangle$ is given, by

$$n_A(\theta) \equiv \langle 0(\theta) | A^\dagger A | 0(\theta) \rangle = \sinh^2(\theta) \quad ; \quad (4.17)$$

and similarly for the modes of type B .

We also observe that the commutativity of J_2 with K_2 ensures that

$$A^\dagger B^\dagger - AB = A^\dagger(\theta) B^\dagger(\theta) - A(\theta) B(\theta). \quad (4.18)$$

By using (4.5), (4.14) and (4.15) we also obtain

$$\mathcal{H}_0 + \mathcal{H}_{I_1} + \mathcal{H}_{I_2} = e^{-i\theta K_2} \mathcal{H}'_0 e^{i\theta K_2} + \mathcal{H}_{I_2} =$$

$$= \hbar\Omega(0)(A^\dagger(\theta)A(\theta) - B^\dagger(\theta)B(\theta)) + i\hbar\Gamma(A^\dagger(\theta)B^\dagger(\theta) - A(\theta)B(\theta)) , \quad (4.19)$$

We have (at finite volume V)

$$|0(\theta, t)\rangle = \frac{1}{\cosh(\Gamma t)} \exp(\tanh(\Gamma t)J_+(\theta))|0(\theta)\rangle , \quad (4.20)$$

with $J_+(\theta) \equiv A^\dagger(\theta)B^\dagger(\theta)$, namely a $su(1, 1)$ generalized coherent state (a two mode Glauber-type coherent state) with equal numbers of modes $A(\theta)$ and $B(\theta)$ condensed in it (for each \mathbf{k}) at each t . At time t $|0(\theta, t)\rangle$ is thus a proper squeezed state. We observe that

$$\langle 0(\theta, t)|0(\theta, t)\rangle = 1 \quad \forall t , \quad (4.21)$$

$$\langle 0(\theta, t)|0(\theta)\rangle = \exp(-\ln \cosh(\Gamma t)) ; \quad (4.22)$$

which shows how, provided $\Gamma > 0$,

$$\langle 0(\theta, t)|0(\theta)\rangle \propto \exp(-t\Gamma) \rightarrow 0 \quad \text{for large } t. \quad (4.23)$$

Thus eq. (4.23) shows the vacuum instability: time evolution brings out of the initial-time Hilbert space for large t . This is not acceptable in quantum mechanics since there the Von Neumann theorem states that all the representations of the canonical commutation relations are unitarily equivalent and therefore there is no room in quantum mechanics for non-unitary time evolution as the one in (4.23). On the contrary, in QFT there exist infinitely many unitarily inequivalent representations and this suggests to us to consider our problem in the framework of QFT, which we will do in the next section.

5 Quantum field theory of inflationary evolution

To set up the formalism in QFT we have to consider the infinite volume limit; however, as customary, we will work at finite volume and at the end of the computations we take the limit $V \rightarrow \infty$. The QFT Hamiltonian is introduced as

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{I_1} + \mathcal{H}_{I_2} \quad (5.1)$$

$$\mathcal{H}_0 = \sum_{\mathbf{k}} \frac{1}{2} \hbar \Omega_{0,\mathbf{k}}(t) (A_{\mathbf{k}}^\dagger A_{\mathbf{k}} - B_{\mathbf{k}}^\dagger B_{\mathbf{k}}) = \sum_{\mathbf{k}} \hbar \Omega_{0,\mathbf{k}}(t) \mathcal{C}_{\mathbf{k}} = \sum_{\mathbf{k}} \hbar \Omega_{0,\mathbf{k}}(t) K_{0,\mathbf{k}} , \quad (5.2)$$

$$\mathcal{H}_{I_1} = - \sum_{\mathbf{k}} \frac{1}{4} \hbar \Omega_{1,\mathbf{k}}(t) \left[(A_{\mathbf{k}}^2 + A_{\mathbf{k}}^{\dagger 2}) - (B_{\mathbf{k}}^2 + B_{\mathbf{k}}^{\dagger 2}) \right] = - \sum_{\mathbf{k}} \hbar \Omega_{1,\mathbf{k}}(t) K_{1,\mathbf{k}} , \quad (5.3)$$

$$\mathcal{H}_{I_2} = i \sum_{\mathbf{k}} \Gamma_{\mathbf{k}} \hbar (A_{\mathbf{k}}^\dagger B_{\mathbf{k}}^\dagger - A_{\mathbf{k}} B_{\mathbf{k}}) = i \sum_{\mathbf{k}} \hbar \Gamma_{\mathbf{k}} (J_{+,\mathbf{k}} - J_{-,\mathbf{k}}) . \quad (5.4)$$

Notice that we have used \mathbf{k} -dependent Γ and the relation between the $\Gamma_{\mathbf{k}}$'s and $\Gamma \equiv \frac{H}{2}$ will be discussed below.

We also have now

$$\mathcal{H}'_0 = \sum_{\mathbf{k}} \hbar \Omega_{\mathbf{k}}(t) (A_{\mathbf{k}}^\dagger A_{\mathbf{k}} - B_{\mathbf{k}}^\dagger B_{\mathbf{k}}) . \quad (5.5)$$

At finite volume V , we formally have

$$|0(\theta, t)\rangle = \prod_{\mathbf{k}} \frac{1}{\cosh(\Gamma_{\mathbf{k}} t)} \exp(\tanh(\Gamma_{\mathbf{k}} t) J_{\mathbf{k},+}(\theta)) |0(\theta)\rangle , \quad (5.6)$$

which corresponds to eq.(4.20). The state $|0(\theta, t)\rangle$ is also a $su(1, 1)$ generalized coherent state. Eqs. (4.21)-(4.23) are now replaced by

$$\langle 0(\theta, t) | 0(\theta, t) \rangle = 1 \quad \forall t , \quad (5.7)$$

$$\langle 0(\theta, t) | 0(\theta) \rangle = \exp\left(-\sum_{\mathbf{k}} \ln \cosh(\Gamma_{\mathbf{k}} t)\right) ; \quad (5.8)$$

which again exhibit non-unitary time evolution, provided $\sum_{\mathbf{k}} \Gamma_{\mathbf{k}} > 0$:

$$\langle 0(\theta, t) | 0(\theta) \rangle \propto \exp\left(-t \sum_{\mathbf{k}} \Gamma_{\mathbf{k}}\right) \rightarrow 0 \quad \text{for large } t . \quad (5.9)$$

Use of the customary continuous limit relation $\sum_{\mathbf{k}} \mapsto \frac{V}{(2\pi)^3} \int d^3\mathbf{k}$, for $\int d^3\mathbf{k} \ln \cosh(\Gamma_{\mathbf{k}} t)$ finite and positive, gives in the infinite volume limit

$$\langle 0(\theta, t) | 0(\theta) \rangle \xrightarrow{V \rightarrow \infty} 0 \quad \forall t , \quad (5.10)$$

$$\langle 0(\theta, t) | 0(\theta', t') \rangle \xrightarrow{V \rightarrow \infty} 0 \quad \text{with } \theta' \equiv \theta(t'_0), \quad \forall t, t', t'_0, \quad t \neq t' . \quad (5.11)$$

We conclude that in the infinite volume limit, vacua at t and at t' , $\forall t, t'$, with $t \neq t'$, are orthogonal and the corresponding Hilbert spaces are unitary inequivalent representations of the canonical commutation relations [21] (see also [31]).

Under time evolution generated by \mathcal{H}_{I_2} the operators $A_{\mathbf{k}}(\theta)$ and $B_{\mathbf{k}}(\theta)$ transform as

$$A_{\mathbf{k}}(\theta) \mapsto A_{\mathbf{k}}(\theta, t) = e^{-i\frac{t}{\hbar} \mathcal{H}_{I_2}} A_{\mathbf{k}}(\theta) e^{i\frac{t}{\hbar} \mathcal{H}_{I_2}} = A_{\mathbf{k}}(\theta) \cosh(\Gamma_{\mathbf{k}} t) - B_{\mathbf{k}}^\dagger(\theta) \sinh(\Gamma_{\mathbf{k}} t) , \quad (5.12)$$

$$B_{\mathbf{k}}(\theta) \mapsto B_{\mathbf{k}}(\theta, t) = e^{-i\frac{t}{\hbar} \mathcal{H}_{I_2}} B_{\mathbf{k}}(\theta) e^{i\frac{t}{\hbar} \mathcal{H}_{I_2}} = -A_{\mathbf{k}}^\dagger(\theta) \sinh(\Gamma_{\mathbf{k}} t) + B_{\mathbf{k}}(\theta) \cosh(\Gamma_{\mathbf{k}} t) . \quad (5.13)$$

These transformations are the Bogoliubov transformations and they can be understood as inner automorphism for the algebra $su_{\mathbf{k}}(1, 1)$ and are canonical as they preserve the commutation relations (3.37). Thus, at every t we have a copy $\{A_{\mathbf{k}}(\theta, t)$,

$A_{\mathbf{k}}^\dagger(\theta, t), B_{\mathbf{k}}(\theta, t), B_{\mathbf{k}}^\dagger(\theta, t); |0(\theta, t) \rangle = |\forall \mathbf{k}\rangle$ of the original algebra and of its highest weight vector $\{A_{\mathbf{k}}(\theta), A_{\mathbf{k}}^\dagger(\theta), B_{\mathbf{k}}(\theta), B_{\mathbf{k}}^\dagger(\theta); |0(\theta) \rangle = |\forall \mathbf{k}\rangle$, induced by the time evolution operator, i.e. we have a realization of the operator algebra at each time t (which can be implemented by Gel'fand-Naimark-Segal construction in the C^* -algebra formalism [29]). The time evolution operator therefore acts as a generator of the group of automorphisms of $\bigoplus_{\mathbf{k}} su_{\mathbf{k}}(1, 1)$ parametrized by time t . We stress that the copies of the original algebra provide unitarily inequivalent representations of the canonical commutation relations in the infinite-volume limit, as shown by eqs. (5.11).

At each time t one has

$$A_{\mathbf{k}}(\theta, t)|0(\theta, t) \rangle = 0 = B_{\mathbf{k}}(\theta, t)|0(\theta, t) \rangle, \quad \forall t, \quad (5.14)$$

and the number of modes of type $A_{\mathbf{k}}(\theta)$ in the state $|0(\theta, t) \rangle$ is given, at each instant t by

$$n_{A_{\mathbf{k}}}(t) \equiv \langle 0(\theta, t) | A_{\mathbf{k}}^\dagger(\theta) A_{\mathbf{k}}(\theta) | 0(\theta, t) \rangle = \langle 0(t) | A_{\mathbf{k}}^\dagger A_{\mathbf{k}} | 0(t) \rangle = \sinh^2(\Gamma_{\mathbf{k}} t) \quad ; \quad (5.15)$$

and similarly for the modes of type $B_{\mathbf{k}}(\theta)$. The state $|0(t) \rangle$ (see eq.(4.13)) has an expression similar to (5.6) with θ missing.

We also observe that the commutativity of \mathcal{C} (i.e. K_0) with \mathcal{H}_{I_2} (i.e. J_2) ensures that the number $(n_{A_{\mathbf{k}}} - n_{B_{\mathbf{k}}})$ is a constant of motion for any \mathbf{k} and any θ . Moreover, one can show [21, 16] that the creation of a mode $A_{\mathbf{k}}(\theta, t)$ is equivalent to the destruction of a mode $B_{\mathbf{k}}(\theta, t)$ and vice-versa. This means that the $B_{\mathbf{k}}(\theta, t)$ modes can be interpreted as the *holes* for the modes $A_{\mathbf{k}}(\theta, t)$: the B -system can be considered as the sink where the energy dissipated by the A -system flows.

Let us now observe that $|0(\theta) \rangle$ is given by

$$|0(\theta) \rangle = \prod_{\mathbf{k}} \frac{1}{\cosh(\theta_{\mathbf{k}})} \exp(\tanh(\theta_{\mathbf{k}}) j_{\mathbf{k},+}) |0 \rangle, \quad (5.16)$$

with $j_{\mathbf{k},+} \equiv a_{\mathbf{k}}^\dagger b_{\mathbf{k}}^\dagger = \frac{1}{2}(A_{\mathbf{k}}^{\dagger 2} + B_{\mathbf{k}}^{\dagger 2})$, and $a_{\mathbf{k}} = \frac{1}{\sqrt{2}}(A_{\mathbf{k}} + iB_{\mathbf{k}})$, $b_{\mathbf{k}} = \frac{1}{\sqrt{2}}(A_{\mathbf{k}} - iB_{\mathbf{k}})$. Thus, $|0(\theta) \rangle$ also is a $su(1, 1)$ generalized coherent state with equal numbers of modes $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ condensed in it for each \mathbf{k} and each t . We thus finally have

$$|0(\theta, t) \rangle = \prod_{\mathbf{k}, \mathbf{q}} \frac{1}{\cosh(\Gamma_{\mathbf{k}} t) \cosh(\theta_{\mathbf{q}})} \exp(\tanh(\Gamma_{\mathbf{k}} t) J_{\mathbf{k},+}(\theta)) \exp(\tanh(\theta_{\mathbf{q}}) j_{\mathbf{q},+}) |0 \rangle. \quad (5.17)$$

In a similar way we could derive the alternative expression for $|0(\theta, t) \rangle$:

$$|0(\theta, t) \rangle = \prod_{\mathbf{k}, \mathbf{q}} \frac{1}{\cosh(\Gamma_{\mathbf{k}} t) \cosh(\theta_{\mathbf{q}})} \exp(\tanh(\theta_{\mathbf{q}}) j_{\mathbf{q},+}(t)) \exp(\tanh(\Gamma_{\mathbf{k}} t) J_{\mathbf{k},+}) |0 \rangle. \quad (5.18)$$

where $J_{\mathbf{k},+} = A_{\mathbf{k}}^\dagger B_{\mathbf{k}}^\dagger$, $j_{\mathbf{q},+}(t) \equiv a_{\mathbf{k}}^\dagger(t) b_{\mathbf{k}}^\dagger(t) = \frac{1}{2}(A_{\mathbf{k}}^{\dagger 2}(t) + B_{\mathbf{k}}^{\dagger 2}(t))$, and $a_{\mathbf{k}}(t) = \frac{1}{\sqrt{2}}(A_{\mathbf{k}}(t) + iB_{\mathbf{k}}(t))$, $b_{\mathbf{k}}(t) = \frac{1}{\sqrt{2}}(A_{\mathbf{k}}(t) - iB_{\mathbf{k}}(t))$, with commutators $[a_{\mathbf{k}}(t), a_{\mathbf{q}}^\dagger(t)] = \delta_{\mathbf{k},\mathbf{q}} = [b_{\mathbf{k}}(t), b_{\mathbf{q}}^\dagger(t)]$ and all other commutators equal to zero. The operators $A_{\mathbf{k}}(t)$ and $B_{\mathbf{k}}(t)$ are given by the (canonical) Bogoliubov transformations

$$A_{\mathbf{k}} \mapsto A_{\mathbf{k}}(t) = e^{-i\frac{t}{\hbar}\mathcal{H}_{I_2}} A_{\mathbf{k}} e^{i\frac{t}{\hbar}\mathcal{H}_{I_2}} = A_{\mathbf{k}} \cosh(\Gamma_{\mathbf{k}} t) - B_{\mathbf{k}}^\dagger \sinh(\Gamma_{\mathbf{k}} t) \quad , \quad (5.19)$$

$$B_{\mathbf{k}} \mapsto B_{\mathbf{k}}(t) = e^{-i\frac{t}{\hbar}\mathcal{H}_{I_2}} B_{\mathbf{k}} e^{i\frac{t}{\hbar}\mathcal{H}_{I_2}} = -A_{\mathbf{k}}^\dagger \sinh(\Gamma_{\mathbf{k}} t) + B_{\mathbf{k}} \cosh(\Gamma_{\mathbf{k}} t) \quad . \quad (5.20)$$

Finally, to establish the relation between the $\Gamma_{\mathbf{k}}$'s and $\Gamma \equiv \frac{H}{2}$ we note that in the continuum limit eqs. (3.37) become

$$[A_{\mathbf{k}}, A_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}') = [B_{\mathbf{k}}, B_{\mathbf{k}'}^\dagger], \quad [A_{\mathbf{k}}, B_{\mathbf{k}'}] = 0, \quad [A_{\mathbf{k}}^\dagger, B_{\mathbf{k}'}^\dagger] = 0 \quad , \quad (5.21)$$

and that, as well known, the $A_{\mathbf{k}}$ (and $B_{\mathbf{k}}$) operators are not well defined on vectors in the Fock space; for instance $|A_{\mathbf{k}}\rangle \equiv A_{\mathbf{k}}^\dagger |0\rangle$ is not a normalizable vector since from eqs. (5.21) one obtains $\langle A_{\mathbf{k}} | A_{\mathbf{k}} \rangle = \delta(\mathbf{0})$ which is infinity. As customary one must then introduce wave-packet (smeared out) operators A_f with spatial distribution described by square-integrable (ortonormal) functions

$$f(\mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3\mathbf{k} f(\mathbf{k}) e^{i\mathbf{k}\mathbf{x}} \quad (5.22)$$

i.e.

$$A_f = \frac{1}{(2\pi)^{3/2}} \int d^3\mathbf{k} A_{\mathbf{k}} f(\mathbf{k}) \quad (5.23)$$

with commutators

$$[A_f, A_g^\dagger] = (f, g) = [B_f, B_g^\dagger], \quad [A_f, B_g] = 0, \quad [A_f^\dagger, B_g^\dagger] = 0 \quad , \quad (5.24)$$

with (f, g) denoting the scalar product between f and g . Now $\langle A_f | A_f \rangle = 1$ and the A_f 's are well defined operators in the Fock space in terms of which the observables have to be expressed. In this connection it is interesting to recall that the reality condition on $\Omega(t)$ (see sec. 3) naturally introduces the infrared cut-off smearing out the operator fields. In conclusion, we express the number operator as

$$n_{A_f}(t) \equiv \langle 0(\theta, t) | A_f^\dagger(\theta) A_f(\theta) | 0(\theta, t) \rangle = \frac{1}{(2\pi)^3} \int d^3\mathbf{k} \sinh^2(\Gamma_{\mathbf{k}} t) |f(\mathbf{k})|^2 \equiv \sinh^2(\Gamma t) \quad , \quad (5.25)$$

and similarly for the modes of type $B_f(\theta)$ (cf. with eq. (5.15)). Eq. (5.25) specifies the relation between the $\Gamma_{\mathbf{k}}$'s and Γ and says that the number of A_f modes does not depend on the volume.

The results obtained in this section clearly show the role of dissipation and its interplay with the time-dependent frequency term in eq. (2.14). By using the conformal time coordinate as usually done in the literature we would not be able to reveal the underlying rich structure of the state space. Such a structure naturally leads us to recognize the thermal properties of $|0(\theta, t)\rangle$. This will be done in the following section.

6 Entropy and free energy in inflating Universe

The vacuum state $|0(\theta, t)\rangle$ as given by equation (5.6) can be written as

$$|0(\theta, t)\rangle = \exp\left(-\frac{1}{2}\mathcal{S}_{A(\theta)}\right)|\mathcal{I}(\theta)\rangle = \exp\left(-\frac{1}{2}\mathcal{S}_{B(\theta)}\right)|\mathcal{I}(\theta)\rangle, \quad (6.1)$$

where

$$|\mathcal{I}(\theta)\rangle \equiv \exp\left(\sum_{\mathbf{k}} A_{\mathbf{k}}^\dagger(\theta) B_{\mathbf{k}}^\dagger(\theta)\right)|0(\theta)\rangle = \exp(-i\theta K_2)|\mathcal{I}\rangle, \quad (6.2)$$

with $|\mathcal{I}\rangle$ the invariant (not normalizable) vector [16]

$$|\mathcal{I}\rangle \equiv \exp\left(\sum_{\mathbf{k}} A_{\mathbf{k}}^\dagger B_{\mathbf{k}}^\dagger\right)|0\rangle, \quad (6.3)$$

and

$$\begin{aligned} \mathcal{S}_{A(\theta)} &\equiv -\sum_{\mathbf{k}} \left\{ A_{\mathbf{k}}^\dagger(\theta) A_{\mathbf{k}}(\theta) \ln \sinh^2(\Gamma_{\mathbf{k}} t) - A_{\mathbf{k}}(\theta) A_{\mathbf{k}}^\dagger(\theta) \ln \cosh^2(\Gamma_{\mathbf{k}} t) \right\} = \\ &= \exp(-i\theta K_2) \mathcal{S}_A \exp(i\theta K_2). \end{aligned} \quad (6.4)$$

Here \mathcal{S}_A is given by

$$\mathcal{S}_A \equiv -\sum_{\mathbf{k}} \left\{ A_{\mathbf{k}}^\dagger A_{\mathbf{k}} \ln \sinh^2(\Gamma_{\mathbf{k}} t) - A_{\mathbf{k}} A_{\mathbf{k}}^\dagger \ln \cosh^2(\Gamma_{\mathbf{k}} t) \right\}. \quad (6.5)$$

$\mathcal{S}_{B(\theta)}$ (\mathcal{S}_B) is given by the same expression with $B_{\mathbf{k}}(\theta)$ ($B_{\mathbf{k}}$) and $B_{\mathbf{k}}^\dagger(\theta)$ ($B_{\mathbf{k}}^\dagger$) replacing $A_{\mathbf{k}}(\theta)$ ($A_{\mathbf{k}}$) and $A_{\mathbf{k}}^\dagger(\theta)$ ($A_{\mathbf{k}}^\dagger$), respectively. In the following we shall simply write $\mathcal{S}(\theta)$ (\mathcal{S}) for either $\mathcal{S}_{A(\theta)}$ or $\mathcal{S}_{B(\theta)}$ (\mathcal{S}_A or \mathcal{S}_B).

We also note that from eq. (6.1) we obtain

$$|0(\theta, t)\rangle = \sum_{n \geq 0} \sqrt{W_n(t)} |n(\theta), n(\theta)\rangle, \quad (6.6)$$

where $n(\theta)$ denotes the multi-index $\{n_{\mathbf{k}}(\theta)\}$, and

$$W_n(t) = \left(\prod_{\mathbf{k}} \frac{\cosh^{2(n_{\mathbf{k}}+1)}(\Gamma_{\mathbf{k}} t)}{\sinh^{2n_{\mathbf{k}}}(\Gamma_{\mathbf{k}} t)} \right)^{-1}, \quad 0 < W_n < 1. \quad (6.7)$$

Notice that the expansion (6.6) contains only terms for which $n_{A_{\mathbf{k}}(\theta)}$ equals $n_{B_{\mathbf{k}}(\theta)}$ for all \mathbf{k} 's and that

$$\sum_{n \geq 0} W_n(t) = 1 \quad \text{for any } t, \quad (6.8)$$

$$\langle 0(\theta, t) | \mathcal{S}(\theta) | 0(\theta, t) \rangle = \langle 0(t) | \mathcal{S} | 0(t) \rangle = - \sum_{n \geq 0} W_n(t) \ln W_n(t). \quad (6.9)$$

Eq. (6.9) leads us therefore to interpreting $\mathcal{S}(\theta)$ as the *entropy* for the dissipative system [21, 16]. We also observe that $\langle 0(\theta, t) | \mathcal{S}(\theta) | 0(\theta, t) \rangle$ grows monotonically with t : the entropy for both A and B increases as the system evolves in time. Moreover, the difference $\mathcal{S}_{A(\theta)} - \mathcal{S}_{B(\theta)}$ is constant in time (cfr. (4.19)):

$$[\mathcal{S}_{A(\theta)} - \mathcal{S}_{B(\theta)}, \mathcal{H}] = 0 \quad (6.10)$$

(and, correspondingly, $[\mathcal{S}_A - \mathcal{S}_B, \mathcal{H}'] = 0$, cfr. (4.5)). Since the B -particles are the holes for the A -particles, $\mathcal{S}_{A(\theta)} - \mathcal{S}_{B(\theta)}$ is in fact the (conserved) entropy for the closed system.

Eqs. (6.1) and (6.4) show that the operational dependence of $\frac{1}{2}\mathcal{S}_{A(\theta)}$ (or respectively, $\frac{1}{2}\mathcal{S}_{B(\theta)}$) is uniquely on the A (B) variables: thus in eq. (6.1) time evolution is expressed solely in terms of the (sub)system A (B) with the elimination of the B (A) variables. This reminds us of the procedure by which one obtains the reduced density matrix by integrating out bath variables.

For the time variation of $|0(\theta, t)\rangle$ at finite volume V , we obtain

$$\frac{\partial}{\partial t} |0(\theta, t)\rangle = -\frac{1}{2} \left(\frac{\partial \mathcal{S}(\theta)}{\partial t} \right) |0(\theta, t)\rangle \quad (6.11)$$

Equation (6.11) shows that $\frac{1}{2} \left(\frac{\partial \mathcal{S}(\theta)}{\partial t} \right)$ is the generator of time-translations, namely time evolution is controlled by the entropy variations. It is remarkable that the operator that controls time evolution is the variation of the dynamical variable whose expectation value is formally an entropy: these features reflect indeed correctly the irreversibility of time evolution characteristic of dissipative (inflating) motion. Dissipation (inflation) implies in fact the choice of a privileged direction in time evolution (*time arrow*) with a consequent breaking of time-reversal invariance.

We conclude that the system in its evolution runs over a variety of representations of the canonical commutation relations which are unitarily inequivalent to each other for $t \neq t'$ in the infinite-volume limit: the non-unitary character of time evolution implied by damping (inflation) is thus recovered, in a consistent scheme, in the unitary inequivalence among representations at different times in the infinite-volume limit.

As already observed in the introduction, the similarities of the above analysis with results on the vacuum structure for *QFT* in curved space-time and on the Hawking radiation for black-hole solutions [9] suggest to us that the doubling of the degrees of freedom is intimately related with the non-trivial metric structure of space-time, the doubled degree of freedom signalling the lost of the Poincaré invariance.

We want now to further analyze the thermal concepts and properties of the formalism above presented.

We observe that the statistical nature of dissipative (inflating) phenomena naturally emerges from our formalism, even though no statistical concepts were introduced a priori (we have seen that the vacuum structure naturally leads to the *entropy operator* as time evolution generator (see eq.(6.11)). We therefore ask ourselves whether

such statistical features may actually be related to thermal concepts. We know from ref. [21] (which here we closely follow) that this is indeed the case.

For the sake of definiteness, let us consider the A -modes alone and introduce the functional

$$F_A \equiv \langle 0(\theta, t) | \left(\mathcal{H}'_{0,A(\theta)} - \frac{1}{\beta} \mathcal{S}_{A(\theta)} \right) | 0(\theta, t) \rangle = \langle 0(t) | \left(\mathcal{H}'_{0,A} - \frac{1}{\beta} \mathcal{S}_A \right) | 0(t) \rangle . \quad (6.12)$$

Here $\mathcal{H}'_{0,A(\theta)} \equiv \sum_{\mathbf{k}} E_{\mathbf{k}} A_{\mathbf{k}}^\dagger(\theta) A_{\mathbf{k}}(\theta)$ and $\mathcal{H}'_{0,A} \equiv \sum_{\mathbf{k}} E_{\mathbf{k}} A_{\mathbf{k}}^\dagger A_{\mathbf{k}}$; β is a strictly positive function of time to be determined and $E_{\mathbf{k}} \equiv \hbar \Omega_{\mathbf{k}}(t_0 = 0) - \mu$, with μ the chemical potential.

We write $\sigma_{\mathbf{k}} \equiv \Gamma_{\mathbf{k}} t$, and look for the values of $\sigma_{\mathbf{k}}$ rendering $F_{A(\theta)}$ stationary:

$$\frac{\partial F_{A(\theta)}}{\partial \sigma_{\mathbf{k}}} = 0 \quad ; \quad \forall \mathbf{k} . \quad (6.13)$$

Condition (6.13) is a stability condition to be satisfied for each representation. We now assume that β is a slowly varying functions of t and thus eq. (6.13) gives

$$\beta E_{\mathbf{k}} = -\ln \tanh^2(\sigma_{\mathbf{k}}) . \quad (6.14)$$

We have then

$$n_{A_{\mathbf{k}}}(t) = \sinh^2(\Gamma_{\mathbf{k}} t) = \frac{1}{e^{\beta(t) E_{\mathbf{k}}} - 1} , \quad (6.15)$$

which is the Bose distribution for $A_{\mathbf{k}}$ at time t provided we assume $\beta(t)$ to represent the inverse temperature $\beta(t) = \frac{1}{k_B T(t)}$ at time t (k_B denotes the Boltzmann constant). This allows us to recognize $\{|0(\theta, t)\rangle\}$ as a representation of the canonical commutation relations at finite temperature, equivalent with the Thermo Field Dynamics representation $\{|0(\beta)\rangle\}$ of Takahashi and Umezawa [16] .

In conclusion we can interpret F_A as the free energy and n_A as the average number of A -modes at the inverse temperature $\beta(t)$ at time t .

The change in time of the energy $E_A \equiv \sum_{\mathbf{k}} E_{\mathbf{k}} n_{A_{\mathbf{k}}}$ is given by

$$dE_A = \frac{\partial}{\partial t} \left(\langle 0(t) | \mathcal{H}'_{0,A} | 0(t) \rangle \right) dt = \sum_{\mathbf{k}} E_{\mathbf{k}} \dot{n}_{A_{\mathbf{k}}}(t) dt \quad ; \quad (6.16)$$

and the change in the entropy by

$$dS_A = \frac{\partial}{\partial t} \left(\langle 0(t) | \mathcal{S}_A | 0(t) \rangle \right) = \beta \sum_{\mathbf{k}} E_{\mathbf{k}} \dot{n}_{A_{\mathbf{k}}}(t) dt = \beta dE_A(t) , \quad (6.17)$$

provided we assume the changes in time of β can be neglected (which happens, *e.g.* in the case of adiabatic variations of temperature, at T high enough) . Thus we have

$$dE_A - \frac{1}{\beta} dS_A = 0 \quad , \quad (6.18)$$

consistently with the relation obtained directly by minimizing the free energy

$$dF_A = dE_A - \frac{1}{\beta} dS_A = 0 \quad . \quad (6.19)$$

Eq. (6.19) expresses the first principle of thermodynamics for a system coupled with environment at constant temperature and in absence of mechanical work and it allows us to recognize E_A as the internal energy of the system. The above discussion shows that time evolution induces transitions over inequivalent representations by inducing changes in the number of condensed modes in the vacuum. By defining as usual heat as $dQ = \frac{1}{\beta} dS$ we see that the change in time \dot{n}_A of particles condensed in the vacuum turns out into heat dissipation dQ [21]. Finally, (6.19) also shows that, provided variations of E_A in the temperature are negligible, entropy is as usual the free energy response to temperature variations.

7 Conclusions

In this paper we have studied the canonical quantization of non-unitary time evolution in inflating Universe. We have shown that the vacuum is a two-mode squeezed state and we have discussed its thermal properties. We have considered only the gravitational wave modes in the FRW metrics in a de Sitter phase. We have shown that the vacuum turns out to be the generalized SU(1,1) coherent state of thermo field dynamics, thus exhibiting the link between inflationary evolution and thermal properties. In particular we have discussed the entropy and the free energy of the system recovering results similar to the ones presented in [14] (see also [42]).

A central ingredient in our discussion has been the doubling of the degrees of freedom, which we have imported from the canonical quantization procedure of the damped harmonic oscillator [21]. The doubling of the degrees of freedom is also a central tool in the TFD formalism of finite temperature QFT, and is thus the bridge to the unified picture of non-unitary time evolution, squeezing and thermal properties in inflating metrics.

From the thermal properties perspective the physical interpretation of the doubled degrees of freedom is the one of the thermal bath degrees of freedom; from the point of view of the vacuum structure the one of *holes* of the relic gravitons; from the hamiltonian formalism point of view the one of the *complement* to the dissipating (inflating) system. In ref. [32] the doubling formalism is shown to be related with the Feynman-Vernon [43] and with the Schwinger formalism [44].

We note that in ref. [14], even if not explicitly stated, the doubling of the degrees of freedom is actually introduced by considering the modes of momentum \mathbf{k} and $-\mathbf{k}$ as the *couple* of modes of total zero momentum in terms of which the two-mode squeezed vacuum is described: the distinction between the \mathbf{k} and $-\mathbf{k}$ modes

introduces a partition in the \mathbf{k} space and leaves out the zero momentum modes which, although not entering in the condensate structures (in [14] the summations are always limited to $\mathbf{k} > 0$), nevertheless are present in the quantized field ϕ and in its canonical momentum π . Also in [6, 7, 8] the doubling is actually present. In fact, in considering the general solution of the parametric oscillator $u(t)$ (we adopt here the same notation of ref. [8]) the authors consider *two* different representations of $u(t)$ in terms of two distinct basis, $\xi(\eta)$ and $\chi(\eta)$, respectively, with η playing the role of time coordinate. In this way the doubling of the degrees of freedom is actually introduced, even if not mentioned. As a matter of fact, the doubling of the degrees of freedom is intrinsic to the Bogoliubov transformations, so that one deals with a doubled system anytime one works with such transformations. For this reason all the "mixed modes" formalisms (since Parker's work [4]) necessarily involve the algebraic structure of the doubling of the modes.

We have shown that the system state space splits into many unitarily inequivalent representations of the canonical commutation relations parametrized by time t , \mathcal{H}_t , and non-unitary time evolution is then described as a trajectory in the space of the representations: the system evolves in time by running over unitarily inequivalent representations. This means that the full set of possible unitarily inequivalent Hilbert spaces must be exploited, which provides further support to known results in QFT in curved space-time [9, 10, 11]. The generator of time evolution is related to the entropy operator, which indeed reflects the irreversibility in time evolution (*the arrow of time*). At the same time, entropy appears as the response of free energy to the temperature variation and thus the intrinsic thermal character of the inflating evolution is exhibited. In this connection, it is interesting to remark that a similar dynamical structure appears in the canonical quantization of matter field in a curved background [9], where the parametrization by proper time of the space of the representations allows the definition of the vacuum state and of the number operator. The present paper discussion and the results of [9] point to the possibility of extending the present canonical quantization scheme also to the case of matter field in the background of expanding Universe. It is then an interesting question to ask how to incorporate in such a scheme the results on the scalar field in an expanding geometry of refs. [45, 46, 47], the self-similar time-dependent scale transformation for the functional Schrödinger equation for a free scalar field [48] and the effective potential models for inflating Universe [49].

As a further remark we would like to point out that the "negative" kinetic term in our Lagrangian structure (cf. section 3) also appears (in the absence of dissipation) in two-dimensional gravity models in conjunction with the problem of the ambiguity of the vacuum definition [50]. In such a situation the choice of the annihilation operators discriminates between two different alternatives: either annihilation operators are associated with positive frequencies and then negative norm states appears, either annihilation operators competing to the positive kinetic term are associated to posi-

tive frequencies and annihilation operators competing to the negative kinetic term are associated to negative frequencies. In this last case no negative norm states appear and the canonical structure is similar to the one of quantum dissipation [21, 36] and of the inflationary time evolution described in the present paper.

Finally, we note that the algebraic structure of the doubling formalism can be related with the quantum deformation of the Weyl-Heisenberg algebra and that, as already mentioned, the TFD algebra is included in the deformed Hopf algebra. We thus expect that the quantum deformation mechanism plays a non-trivial role in the quantization procedure in expanding geometry. In particular, we expect [33] that the deformation q -parameter is related with the inflating constant (the Hubble constant).

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